Displacement Interaction and Surface Curvature Effects on Hypersonic Boundary Layers

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Theme

At high-flight Mach numbers displacement interaction and longitudinal surface curvature effects—normally considered to be of higher order in boundary-layer theory—may become significant even for flows with large Reynolds numbers. It has been speculated that these effects might produce marked changes in surface pressures and heat transfer. Thus a numerical means to assess the importance of these so-called higher-order effects is needed and such a method is demonstrated here.

Contents

The governing viscous equations employed here are the composite second-order laminar compressible boundary-layer equations formulated by Davis, Werle, and Wornom.[1] In that formulation, no attempt was made to separate what are normally termed first-order and second-order boundary-layer effects; there, all such terms were included in a single set of equations. While this formulation is mathematically valid only to second order, it is hoped that some advantage might be gained from its application to cases where displacement and surface curvature effects are not small. To the authors' knowledge, the solutions presented here represent the first nonsimilar solutions to such a formulation.

Before solving, the governing equations are transformed by generalized forms of the Görtler and Howarth-Dorodnitsyn transformations to an ξ, η plane where the boundary-layer growth is retarded and the initial similarity solution required is readily obtainable. Numerical solutions are then obtained by an implicit finite-difference method similar to that developed by Flügge-Lotz and Blottner[2] with later modifications by Davis.[3]

Of the effects studied here, the effect of surface curvature is included explicitly in the governing equations and therefore presented no unusual computational difficulty. To incorporate the displacement effect into the solution, that is, to develop a scheme for simultaneously solving the boundary layer and its effect on the inviscid streamline displacement, an iterative scheme similar to that of Ref. 2 was employed. Examination of the viscous governing equations shows that once the displacement thickness and its first and second derivatives with respect to the surface distance s are known, all inviscid quantities necessary to compute the boundary-layer solution can be calculated. The over-all method of solution can be summarized as follows: 1) an initial guess of the displacement thickness δ is made from which the necessary inviscid properties are computed using the tangent-wedge rule[4] and one iteration made on the boundary-layer solution; 2) using these boundary-layer profiles a new estimate of δ is obtained; 3) employing backward difference formulas, new estimates of the displacement derivatives δ' and δ'' are made; and 4) the above procedure is repeated until the solution has converged within some acceptable error. Boundary-layer profiles and inviscid properties at the station where the interaction was initiated were obtained by a modified form of the classical strong-interaction boundary-layer model.[4]

The preceding procedure was applied to the case of two-dimensional laminar flow up a cubic compression ramp given by $y = x^3/150$ (the reference length was taken to be 1 in.) placed in a hypersonic mainstream. Figure 1 shows the ratio of the pressure in the viscous region to the freestream value as a function of the nondimensional normal boundary-layer coordinate N. This figure shows the second-order solution pressure profile which includes both displacement and longitudinal surface curvature effects along with the classical or first-order prediction—both at the $s = 1.5$ station. It is observed that the variation between the second-order wall pressure and the inviscid surface pressure is approximately 50%. However, the actual pressure in the boundary layer is found to be approximately constant over most of the viscous region, and this value is quite close to the classical or first-order estimate. Thus, in the presence of displacement interaction, the inviscid and viscous curvature corrections apparently offset each other for this case. This is verified in Fig. 2 which shows the computed wall pressure for the previous case along with the approximate solution of Stollery[5] which includes displacement effects only. Also shown are the experimental data of Stollery[5] and the classical or first-order prediction. Over most of the surface, the present solution shows an appreciable difference from Stollery's displacement-only solution except near the leading edge where the curvature is very small.

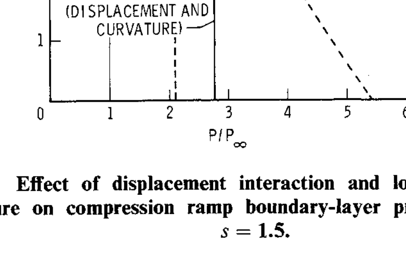

Fig. 1 Effect of displacement interaction and longitudinal surface curvature on compression ramp boundary-layer pressure profiles at $s = 1.5$.

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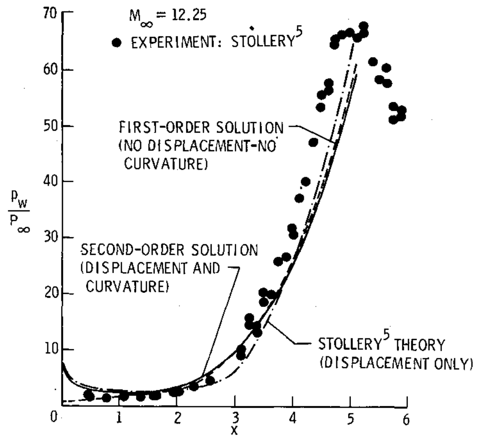

Fig. 2 The influence of displacement body and curvature effects on ramp wall pressure.

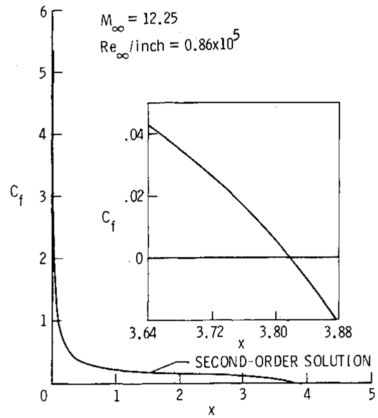

Fig. 3 Separated flow on a cubic compression ramp.

Note, however, the present second-order solution virtually reproduces the first-order result downstream of the leading edge thus indicating that the displacement and curvature effects are offsetting one another. Such behavior was also noted for Mach numbers 6 and 8 and freestream unit Reynolds numbers varying from 0.258×10^5 to 0.860×10^5. It was observed also that when either curvature correction (inviscid or viscous) was omitted, large errors or numerical difficulties were encountered in the computed solutions (see Wornom[6]).

A final test of the technique presented here consisted of a variation on the freestream unit Reynolds number. Of special interest is the solution shown in Fig. 3 which terminated due to separation of the viscous flow from the ramp surface. Figure 3, which gives details of the skin-friction coefficient C_f in the region where it goes to zero, shows no evidence of an impending singularity as the point of zero shear stress is approached; in fact, the computation proceeded downstream of the zero shear-stress point before the numerical method employed to solve the boundary-layer equations became completely unstable due to the reversed flow region near the wall. Thus, even with higher-order curvature corrections, the boundary-layer solutions appear to be regular at separation so long as displacement effects are accounted for.

In conclusion, the effects of displacement thickness interaction and longitudinal surface on hypersonic flow up a two-dimensional cubic compression ramp can be summarized as follows. When a consistent treatment of inviscid and viscous curvature corrections was employed along with the displacement interaction computation, the second-order theory virtually reproduced the classical or first-order results. It was found also that when separation occurs in the presence of displacement thickness interaction and curvature corrections, no singularity is observed in the skin friction at the separation point.

References

[1] Davis, R. T., Werle, M. J. and Wornom, S. F., "A Consistent Formulation of Compressible Boundary-Layer Theory With Second-Order Curvature and Displacement Effects," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1701–1703.

[2] Flügge-Lotz, I. and Blottner, F. G., "Computation of the Compressible Laminar Boundary-Layer Flow Including Displacement Thickness Interaction Using Finite-Difference Methods," TR 131, Jan. 1962, Div. of Engineering Mechanics, Stanford Univ., Stanford, Calif.

[3] Davis, R. T., "Numerical Solution of the Hypersonic Viscous Shock-Layer Equations," *AIAA Journal*, Vol. 8, No. 5, May 1970, pp. 843–851.

[4] Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory*, Academic Press, New York, 1959, pp. 277–281 and 292–306.

[5] Stollery, J. L., "Hypersonic Viscous Interaction on Curved Surfaces," ARL-70-0126, July 1970, Aerospace Research Lab., Wright-Patterson Air Force Base, Ohio.

[6] Wornom, S. F., "A Numerical Study of Displacement Body on Curvature Effects on Incompressible and Compressible Laminar Boundary-Layers," Ph.D. dissertation, June 1971, Virginia Polytechnic Inst. and State Univ. Blacksburg, Va.